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# On the consistency of a Fermion-Torsion effective theory

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Abstract: We discuss the possibility to construct an effective quantum field theory for an axial vector coupled to a Dirac spinor field. A massive axial vector describes antisymmetric torsion. The consistency conditions include unitarity and renormalizability in the low-energy region. The investigation of the Ward identities and the one- and two-loop divergences indicate serious problems arising in the theory. The final conclusion is that torsion may exist as a string excitation, but there are very severe restrictions for the existence of a propagating torsion field, subject to the quantization procedure, at low energies.

Keywords: Renormalization Regularization and Renormalons, Gauge Symmetry, Models of Quantum Gravity.

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## Contents



## 1. Introduction

With respect to the space-time symmetries, the Standard Model of the Elementary Particle physics includes three types of fields: spinors, vectors and scalars. The same concerns Grand Unified Theories, which are indeed based on larger symmetry groups. The effective interactions of QCD lead to the pion field, which is a pseudo-scalar. One might, naturally, ask whether there may be other fields or interactions which can be unobservable at low energies. This question becomes particularly important in view of the fact that the (super)string theories yield, in their low-energy spectrum, some fields different from the ones mentioned above. Most of these fields are not propagating (and, consequently, are not visible) at available energies, because they have too huge masses (typically of the Planck order). This concerns, at first, the higher-spin excitations related to the massive string modes. Besides, in addition to the usual fields, the massless excitations of the string spectrum contain a skew-symmetric tensor, which eventually produces, in the low-energy effective string action, the 3-form associated to torsion. In known string theories, this tensor shows up at first order in $\alpha'$ and has a mass of the Planck order. Therefore, it doesn't propagate at low energies. However, it is interesting to investigate the possibility



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that this field possesses an essentially smaller (or zero) mass, so that torsion could propagate. This implies the low theoretical bound for the torsion mass.

Here, we take the viewpoint according to which any propagating field must be quantized, so that the classical theory is nothing but an approximation for the complete theory including quantum corrections. Then, the appropriate framework for the investigation of a propagating torsion is the effective quantum field theory approach (see, for example, [1]). From the modern point of view, most of the existing quantum field theories should be regarded as effective ones, descending from some other more fundamental theories. The classical action of the effective theories may have the form of an infinite series whose expansion is performed in the inverse of some large massive parameter. At low energies, only the first terms of the expansion are relevant, so that one can consequently disregard high-derivative terms, though some consistency conditions should be indeed satisfied. In particular, the theory must be unitary and renormalizable in the given low-energy region. For the case of torsion, these consistency conditions have been applied in [2]. It was shown that the theory possesses an extra, softly broken, gauge symmetry and that this symmetry fixes, in a unique way, the form of the low-energy classical action. This action succeeds in the test based on the calculation of the fermion determinant [2] and led to a wide set of phenomenological consequences.

The purpose of the present paper is to proceed further with the study of the possibility to construct a quantum field theory for a fermion-torsion system. In [2], the unique candidate to be torsion action was suggested and some of its theoretical and phenomenological aspects were discussed. It is well known that the theory of axial vector field may have problems, and these problems are usually associated to the axial anomaly. However, in the case of torsion embedding into the Standard Model, the anomaly can not appear due to the algebraic reasons [2], because all the vector ingredients of the SM have group index which is absent for torsion. However, as the example of the scalar-fermion-torsion shows, the absense of anomaly does not guarantee consistency, and in particular the conflict between renormalizability and unitarity takes place. Here we are going to investigate whether the Ward-Takahashi identities and the one- and two-loop divergences arising in the fermion-torsion system are consistent with the requirements an effective quantum field theory should fulfill. This study is necessary for the final answer of whether the space-time torsion can exist as an independent field, propagating at low energies, which is subject of quantization.

Our paper is organized as follows: in section 2, a brief review of the previous results is given and the main purpose of the subsequent study is formulated. Next, in section 3, we discuss in more details the symmetries of the theory, the analogue of Boulware transformation [3] and the Ward identities corresponding to the softly broken symmetry associated to torsion. For pedagogical purposes, we simultaneously state similar considerations for the vector field. Section 4 is devoted to the calculation of the 1-loop divergences, with many technicalities and the calculations for

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three simpler models are postponed to the appendix A. These calculations include the one for the massive vector coupled to fermions. In order to perform calculations for the cases of the massive vector and massive axial vector, we apply the generalized Schwinger-DeWitt technique, developed in [4], which we supplement by some technical tricks. The validity of the calculational method is verified in two massless cases, for which the result may be achieved through the Faddeev-Popov method. Section 5 contains further analysis of the 1-loop renormalization and renormalization group equations. Section 6 is devoted to the evaluation of the leading two-loop divergences. We apply, in this section, the expansion of the loop integrals suggested in [2]. Since the results of these two-loop calculations have great importance for the qualitative output of our study, they are checked in appendix B by using the standard Feynman parameter method. Finally, in section 6, we draw our Conclusions.

## 2. Dynamical torsion: review of previous results

In this section, we briefly present previous results. We start off by the background notions for the gravity with torsion and quantum theory of matter fields in an external torsion field. A pedagogical introduction may be found in [5].

In the space - time with independent metric and torsion, the affine connection $\tilde{\Gamma}^{\alpha}_{\beta\gamma}$ is non-symmetric, and the torsion tensor is defined as $T^{\alpha}_{\beta\gamma} = \tilde{\Gamma}^{\alpha}_{\beta\gamma} - \tilde{\Gamma}^{\alpha}_{\gamma\beta}$. The covariant derivative, $\tilde{\nabla}_{\mu}$, is based on the non-symmetric connection $\tilde{\Gamma}^{\alpha}_{\beta\gamma}$, while the notation $\nabla_{\mu}$ is kept for the riemannian covariant derivative. From the metricity condition, $\tilde{\nabla}_{\mu} g_{\alpha\beta} = 0$, the solution for the connection can be easily found. It proves useful to divide the torsion field into three irreducible components:

$$T_{\alpha\beta\mu} = \frac{1}{3}\left(T_{\beta} g_{\alpha\mu} - T_{\mu} g_{\alpha\beta}\right) - \frac{1}{6}\varepsilon_{\alpha\beta\mu\nu} S^{\nu} + q_{\alpha\beta\mu}\,, \tag{2.1}$$

where the last tensor satisfies the conditions $q^{\alpha}_{\beta\alpha} = 0$ and $\varepsilon^{\alpha\beta\mu\nu} q_{\alpha\beta\mu} = 0$.

Let us now consider the interaction of torsion with matter fields. The interaction between a Dirac field, $\psi$, and an external gravitational field with torsion is described by the action:

$$S_{1/2} = i \int d^4x \bar{\psi}\left[\gamma^{\alpha}\left(\partial_{\alpha} - iqV_{\alpha} + i\eta\gamma_5 S_{\alpha}\right) - im\right]\psi\,, \tag{2.2}$$

where $\eta$ is an arbitrary parameter, which equals $1/8$ for the special case of minimal coupling. For our purposes, it is useful to keep $\eta$ arbitrary. We have included the abelian vector field, $V_{\alpha}$, for the sake of further convenience.

The study of the renormalization of gauge models in an external gravitational field with torsion has been carried out in [6]. In the general case, the theory includes gauge as well as scalar and fermion fields linked by corresponding interactions (typical examples are the Standard Model or GUT's), the non-minimal interaction with torsion proved necessary not only for the spinor, but also for scalar fields. In the last case, the essential (necessary for the renormalizability) interactions are described by

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the action:

$$S_{sc} = \int d^4x \left\{ \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} \xi S_\mu S^\mu \varphi^2 + \frac{1}{2} \xi_1 R \varphi^2 \right\}. \tag{2.3}$$

Here, $\xi, \xi_1$ are non-minimal parameters. One has to notice that only the interactions with the axial vector, $S_\mu$, are important in both cases (2.2) and (2.3). The interaction of scalars with $q^{\alpha}_{\cdot\mu\nu}$ and both spinors and scalars with $T_\mu$ may be introduced, but it is purely non-minimal. In the sequel, we consider only the axial part, $S_\mu$, of the torsion tensor. Also, since metric and torsion are independent fields, and we are especially interested in the torsion effects, in what follows we consider the flat metric only.

The problem of the action for the dynamical torsion field is crucially important for all investigations of the gravity with torsion. In the literature, one can meet several different approaches for the construction of a torsion action [7, 8, 9, 10, 2]. In particular, [7] started from the gauge principle for gravity (similar ideas are very popular; see [11, 12] for a comprehensive review). In [8, 9], the family of the high-derivative metric-torsion actions, leading to theories without unphysical massive spin-2 ghosts has been constructed. Therefore, in these works, the guiding principle was the unitarity of the theory. In the analysis of the physical significance of torsion, its most important part is the axial component, $S_\mu$, for it is the component which couples to the fermions. In [10], it was readily noticed that for the study of the possible torsion effects at low energies, only the second-derivative terms are indeed relevant. Furthermore, in [10], it was established that, as usual for the vector field, the propagation of both the transverse and the longitudinal parts of the axial vector $S_\mu$ unavoidably breaks unitarity (see, for example, [13]). After that, in [10], only the longitudinal part of $S_\mu$ has been considered, and torsion was thereby reduced to its pseudoscalar piece. In [2], the problem of consistency had been formulated in a closed form, taking both aspects of effective field theory into account. The choice of the action for the dynamical torsion field should be made in such a way that it leads to a unitary and renormalizable effective quantum field theory.

Let us see how this principle can be applied to the fermion-torsion interaction. Starting from (2.2), we may notice that this action possesses two symmetries: the usual gauge one,

$$\psi' = \psi e^{i\alpha(x)}, \qquad \bar{\psi}' = \bar{\psi} e^{-i\alpha(x)}, \qquad V'_\mu = V_\mu + q^{-1} \partial_\mu \alpha(x) \tag{2.4}$$

and an additional symmetry which is softly broken by the spinor mass [2, 14]:

$$\psi' = e^{i\gamma_5 \beta(x)} \psi, \qquad \bar{\psi}' = \bar{\psi} e^{i\gamma_5 \beta(x)}, \qquad S'_\mu = S_\mu - \eta^{-1} \partial_\mu \beta(x). \tag{2.5}$$

In fact, the last symmetry is the key point allowing to set up a unique form of the torsion action. Even softly broken, this symmetry yields the appearance of the transverse second-derivative counterterm $S^2_{\mu\nu}$ and (exactly because it is softly broken) the massive counterterm, both coming, for instance, from a single fermion loop (see, for example, our calculations in the next section). Thus, if we wish to



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have a renormalizable effective field theory for the torsion, these two terms must be included into the action for a dynamical torsion. On the other hand, the condition of unitarity forbids the third possible structure,[1] $(\partial_\mu S^\mu)^2$.

Therefore, the only chance to meet the conditions for the low-energy renormalizability and unitarity is to choose the expression

$$S_{tor-fer} = \int d^4x \left\{ -\frac{1}{4} S_{\mu\nu} S^{\mu\nu} + \frac{1}{2} M^2 S_\mu S^\mu + i\bar{\psi}[\gamma^\alpha(\partial_\alpha + i\eta\gamma_5 S_\alpha) - im]\psi \right\} \tag{2.6}$$

as the torsion-fermion action.

Expression (2.6) shall be the main object of study in the present paper. However, it is very instructive for us to see, how the introduction of the scalar fields explicitly breaks the above scheme. One can consult the second work of ref. [2] for a complete consideration. When one implements scalars, the Yukawa interaction produces a rigid breaking of the symmetry (2.5). This happens because the Yukawa coupling is massless. As a result of this breaking, there are no restrictions on the divergences coming from the diagrams including the Yukawa vertex. As it was proved in [2], these diagrams really require the longitudinal counterterm $(\partial_\alpha S^\alpha)^2$ at the two-loop level. Of course, in order to have a renormalizable theory, the term $(\partial_\mu S^\mu)^2$ might be introduced into the torsion action but, as it was already mentioned, this immediately breaks unitarity. Therefore, in the torsion-fermion-scalar theory, there is a manifest conflict between renormalizability and unitarity. This conflict resembles the similar one which takes place in high-derivative gravity [15, 5]. The difference is that, for gravity, there is a massless mode which provides classical effects through the propagation of graviton, while for the torsion there are no massless modes, and if the lightest torsion mode has a mass of the Planck order, then an independent torsion field simply does not exist.

One can imagine several possibilities to overcome the crisis between renormalizability and unitarity, as described above. For instance, it is possible to search for an extra symmetry providing the cancellation of the longitudinal divergences. Another option is to restrict our considerations to theories without fundamental scalars, such as Technicolour or the Nambu-Jona-Lasinho models. In view of this, it becomes especially important to investigate whether the fermion-torsion system satisfies the consistency conditions. In the present paper, we are going to make a complete study of the conditions of renormalizability and unitarity for the fermion-torsion system without scalar fields.

In the next sections, we shall show that, unfortunately, despite the breaking of the symmetry (2.5) is soft, the final situation is very similar to the one with the scalar fields. One may maintain the unitarity of the renormalized theory, but only at the expenses of a very rigid limit on the torsion mass, which must be much larger than the one of the fermions and much lighter than the fundamental scale.

[1] All other possible terms exhibit higher derivatives or they are non-local.

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## 3. Boulware's parametrization and the Ward identities

We need to perform an analogue of Boulware transformation [3] in the fermion - axial vector system. For pedagogical reasons, we first consider the usual vector case, that is, repeat the transformation of [3]. The action, in original variables, has the form:

$$S_{m-vec} = \int d^4x \left\{ -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} M^2 V_\mu V^\mu + i\bar{\psi}[\gamma^\alpha(\partial_\alpha - igV_\alpha) - im]\psi \right\}, \tag{3.1}$$

and, after the change of the field variables [3]:

$$\psi = \exp\left\{\frac{ig}{M}\cdot\varphi\right\}\cdot\chi\,, \qquad \bar{\psi} = \bar{\chi}\cdot\exp\left\{-\frac{ig}{M}\cdot\varphi\right\}, \qquad V_\mu = V_\mu^\perp - \frac{1}{M}\partial_\mu\varphi\,, \tag{3.2}$$

the new scalar, $\varphi$, is completely factored out:

$$S_{m-vec} = \int d^4x \left\{ -\frac{1}{4}\left(V_{\mu\nu}^\perp\right)^2 + \frac{1}{2}M^2 V_\mu^\perp V^{\perp\mu} + \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi + i\bar{\chi}[\gamma^\alpha(\partial_\alpha + igV_\alpha^\perp)]\chi \right\}. \tag{3.3}$$

Let us now consider the fermion-torsion system given by the action (2.6). The change of variables, similar to the one in (3.2), has the form:

$$\psi = \exp\left\{\frac{i\eta}{M}\gamma^5\varphi\right\}\chi\,, \qquad \bar{\psi} = \bar{\chi}\,\exp\left\{\frac{i\eta}{M}\gamma^5\varphi\right\}, \qquad S_\mu = S_\mu^\perp - \frac{1}{M}\partial_\mu\varphi\,, \tag{3.4}$$

where $S_\mu^\perp$ and $S_\mu^{||} = \partial_\mu\varphi$ are the transverse and longitudinal parts of the axial vector respectively, the latter being equivalent to the pseudoscalar $\varphi$. One has to notice that, contrary to (3.2), but in full accordance with (2.5), the signs of both the exponents in (3.4) are the same. In terms of the new variables, the action becomes

$$S_{tor-fer} = \int d^4x \left\{ -\frac{1}{4} S_{\mu\nu}^\perp S^{\perp\mu\nu} + \frac{1}{2}M^2 S_\mu^\perp S^{\perp\mu} + \right.$$
$$\left. + i\bar{\chi}[\gamma^\alpha(\partial_\alpha + i\eta\gamma_5 S_\alpha^\perp) - im\cdot e^{\frac{2i\eta}{M}\gamma^5\varphi}]\chi + \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi \right\}, \tag{3.5}$$

where $S_{\mu\nu}^\perp = \partial_\mu S_\nu^\perp - \partial_\mu S_\nu^\perp = S_{\mu\nu}$. The last expression can be more easily analyzed by comparison with a similar parametrization for the massive abelian field (3.2). Contrary to the last, for the torsion axial vector (3.5) the scalar mode does not decouple, but rather couple with interactions as follows:

(i) Yukawa-type, resembling the problems with the ordinary scalar.

(ii) Exponential, which prevents the model from being power-counting renormalizable.



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However, at first sight, there is a hope that the above features would not be fatal for the theory. With respect to the point (i), one can guess that the only result of the non-factorization, which could be dangerous for the consistency of the effective quantum theory, would be the propagation of the longitudinal mode of the torsion, and this does not directly follow from the non-factorization of the scalar degree of freedom in the classical action. On the other hand, (ii) indicates the non-renormalizability, which might mean just the appearance of the higher-derivative divergences, that do not matter within the effective approach. Thus, a more detailed analysis is necessary. In particular, the one-loop calculation in the theory (2.6) may be helpful, and it will be done in the next section.

Let us consider, for a moment, the Ward-Takahashi identities for the two theories (3.1) and (2.6.) In the case of the massive vector (3.1), the identity for the effective action, $\Gamma[V_\mu, \psi, \bar{\psi}]$, (here $V_\mu, \psi, \bar{\psi}$ are mean, or background fields) has the form

$$i\partial_\mu \frac{\delta \Gamma}{\delta V_\mu} + ie\left(\bar{\psi}\frac{\delta \Gamma}{\delta \bar{\psi}} - \frac{\delta \Gamma}{\delta \psi}\psi\right) - iM^2 \partial^\mu V_\mu = 0\,. \tag{3.6}$$

Applying the $\delta/\delta V_\mu$ operator, and setting $V_\mu, \psi, \bar{\psi} = 0$, we get the identities for the inverse propagator

$$\partial_\mu \frac{\delta^2 \Gamma}{\delta V_\mu(x) \delta V_\nu(y)} = M^2 \partial_\nu \delta(x-y)\,. \tag{3.7}$$

Now, applying $\delta^2/\delta\psi(y)\delta\bar{\psi}(z)$, one obtains

$$\partial_\mu \frac{\delta^3 \Gamma}{\delta\psi(y)\delta\bar{\psi}(z)\delta V_\mu(x)} = ie\left(\frac{\delta^2 \Gamma}{\delta\psi(y)\delta\bar{\psi}(z)}\delta(x-y) - \frac{\delta^2 \Gamma}{\delta\psi(y)\delta\bar{\psi}(z)}\delta(x-z)\right). \tag{3.8}$$

Similar relations take place for other vertices. The vector mass completely decouples and shows up exclusively in the propagator. Indeed, under these circumstances, it cannot affect the divergences, except in some trivial way. The result is nothing but the direct confirmation of the decoupling which is observed in Boulware-like parametrization (3.3).

In our case of the massive axial vector field (2.6), we have, by means of analogous procedures,

$$-\partial_\mu \frac{\delta \Gamma}{\delta S_\mu} - i\eta\left(\bar{\psi}\gamma_5 \frac{\delta \Gamma}{\delta \bar{\psi}} - \frac{\delta \Gamma}{\delta \psi}\gamma_5 \psi\right) + 2i\eta m \bar{\psi}\gamma_5\psi + M^2 \partial^\mu S_\mu = 0\,. \tag{3.9}$$

Applying functional derivatives, the Ward-Takahashi identities for the inverse propagator and the vertices take over the form:

$$\partial_\mu \frac{\delta^2 \Gamma}{\delta S_\mu(x) \delta S_\nu(y)} = M^2 \partial_\nu \delta(x-y) \tag{3.10}$$

and

$$\begin{aligned}\partial_\mu \frac{\delta^3 \Gamma}{\delta\bar{\psi}(z)\delta S_\mu(x)\delta\psi(y)} &= -2i\eta m \gamma_5 \delta(x-y)\delta(x-z) + \\ &+ i\eta\left(-\frac{\delta^2 \Gamma}{\delta\bar{\psi}(z)\delta\psi(y)}\gamma_5\delta(x-y) + \delta(x-z)\gamma_5 \frac{\delta^2 \Gamma}{\delta\psi(y)\delta\bar{\psi}(z)}\right),\end{aligned} \tag{3.11}$$



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and so on. The last expressions manifest the clear difference with respect to the previous ones, (3.6)–(3.8). In the axial vector case, the massive term affects the interaction vertices, and one can expect that some non-invariant divergences may show up.

## 4. One-loop calculation in the fermion-torsion case

The purpose of this section is to derive the full set of 1-loop counterterms for the massive axial vector coupled to the Dirac spinor. To get them, we are going to apply the background field method together with the Schwinger-DeWitt expansion. However, since the use of these methods for the system of interest is quite non-trivial, and also for pedagogical reasons, we perform also three auxiliary calculations: for the massless vector coupled to a massive spinor (QED), for the massless axial vector (this one coupled to massless spinors) and for the massive vector, all using the same calculational scheme as for the case of the massive axial vector. These additional calculations are collected in the appendix A. Here, we present the details of calculation for the massive axial vector.

Let us start from the fermionic determinant, which was already considered by many authors [16, 17, 18, 2]. The contribution from the single fermion loop is given by the expression

$$\Gamma^{(1)}_{\text{fermion}} = -i\,\mathrm{Tr}\ln\hat{H}\,, \qquad \text{where} \qquad \hat{H} = \{i\gamma^\mu D_\mu + m\}\,. \tag{4.1}$$

Here, $D_\mu = \partial_\mu + i\eta\gamma^5 S_\mu$ is the covariant derivative. It proves useful to introduce the conjugate derivative, $D^*_\mu = \partial_\mu - i\eta\gamma^5 S_\mu$. Then, one can write

$$\begin{aligned}\Gamma_{\text{fermion}} &= -\frac{i}{2}\,\mathrm{Tr}\ln\hat{H}\cdot\hat{H}^* = -\frac{i}{2}\,\mathrm{Tr}\ln\{-\gamma^\mu D_\mu\gamma^\nu D_\nu - m^2\}\\ &= -\frac{i}{2}\,\mathrm{Tr}\ln\{-(\gamma^\mu\gamma^\nu D^*_\mu D_\nu + m^2)\}\,.\end{aligned} \tag{4.2}$$

After a simple algebra, one can cast two useful forms for the operator between parenthesis: the non-covariant:

$$-\hat{H}\cdot\hat{H}^* = \partial^2 + R^\mu\partial_\mu + \Pi,$$

with

$$R^\mu = 2\eta\sigma^{\mu\nu}\gamma_5 S_\nu\,, \qquad \Pi = i\eta\gamma_5(\partial_\mu S^\mu) + \frac{i}{2}\eta\gamma^\mu\gamma^\nu\gamma_5 S_{\mu\nu} + \eta^2 S_\mu S^\mu + m^2 \tag{4.3}$$

and covariant

$$-\hat{H}\cdot\hat{H}^* = D^2 + E^\mu D_\mu + F\,, \tag{4.4}$$

with

$$E^\mu = 2\eta\sigma^{\mu\nu}\gamma_5 S_\nu - 2i\eta\gamma^5 S^\mu\,, \qquad F = m^2 + \frac{i}{2}\eta\gamma^\mu\gamma^\nu\gamma^5 S_{\mu\nu}\,. \tag{4.5}$$



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Both expressions are compatible with the use of the standard Schwinger-DeWitt technique (covariant calculation is much shorter), which yields the well-known result

$$\Gamma^{(1)}_{\text{fermion,div}} = \frac{\mu^{n-4}}{\varepsilon} \int d^n x \left\{ \frac{2\eta^2}{3} S_{\mu\nu} S^{\mu\nu} - 8m^2\eta^2 S^\mu S_\mu + 2m^4 \right\}. \tag{4.6}$$

Here, $\varepsilon = (4\pi)^2(n-4)$ is the parameter of dimensional regularization.

Now, we are in a position to start the complete calculation of divergences. The use of the background field method supposes the split (shift) of the field variables into a background and a quantum part. However, in the case of the (axial)vector-fermion system, the simple shift of the fields leads to an enormous volume of calculations, even for a massive vector. Such a calculation becomes extremely difficult for the axial massive vector (2.6). That is why we have invented a simple trick combining the background field method with the Boulware transformation (3.4) for the quantum fields. As we shall see in a moment, our method makes the calculations reasonably simpler.

Let us divide the fields into background $(S_\mu, \psi, \bar\psi)$ and quantum $(t^\perp_\mu, \varphi, \chi, \bar\chi)$ parts, according to what follows:

$$\begin{aligned}
\psi \to \psi' &= e^{i\frac{\eta}{M}\gamma_5\varphi} \cdot (\psi + \chi), \\
\bar\psi \to \bar\psi' &= (\bar\psi + \bar\chi) \cdot e^{i\frac{\eta}{M}\gamma_5\varphi}, \\
S_\mu \to S'_\mu &= S_\mu + t^\perp_\mu - \frac{1}{M}\partial_\mu\varphi .
\end{aligned} \tag{4.7}$$

The one-loop effective action depends on the quadratic (in quantum fields) part of the total action:

$$\begin{aligned}
S^{(2)} = \frac{1}{2}\int d^4x \Big\{ & t^\perp_\mu(\Box + M^2)t^{\perp\mu} + \varphi(-\Box)\varphi + t^\perp_\mu(-2\eta\bar\psi\gamma^\mu\gamma_5)\chi + \\
& + \varphi\left(-\frac{4m\eta^2}{M^2}\bar\psi\psi\right)\varphi + \bar\chi(-2\eta\gamma^\nu\gamma^5\psi)t^\perp_\nu + \bar\chi\left(\frac{4im\eta}{M}\gamma^5\psi\right)\varphi + \\
& + \varphi\left(\frac{4i\eta m}{M}\bar\psi\gamma^5\right)\chi + \bar\chi(2i\gamma^\mu D_\mu + 2m)\chi \Big\}.
\end{aligned} \tag{4.8}$$

Making the usual change of the fermionic variables,[2] $\chi = -\frac{i}{2}(\gamma^\mu D_\mu + im)\tau$, and substituting $\varphi \to i\varphi$, we arrive at the following expression:

$$S^{(2)} = \frac{1}{2}\int d^4x \, (\, t^\perp_\mu \quad \varphi \quad \bar\chi\,) \cdot \hat{\mathbf{H}} \cdot \begin{pmatrix} t^\perp_\nu \\ \varphi \\ \tau \end{pmatrix},$$

where the hermitean bilinear form $\hat{\mathbf{H}}$ has the form

$$\hat{\mathbf{H}} = \begin{pmatrix} \theta^{\mu\nu}(\Box + M^2) & 0 & \theta^\mu{}_\beta(L^{\beta\alpha}\partial_\alpha + M^\beta) \\ 0 & \Box + N & A^\alpha\partial_\alpha + B \\ P_\beta\theta^{\beta\nu} & Q & \hat{1}\Box + R^\lambda\partial_\lambda + \Pi \end{pmatrix}, \tag{4.9}$$

[2] One has to remember that the jacobian of this change of variables has been already taken into account before, and its divergences were counted in (4.6).

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$\theta^\mu{}_\nu = \delta^\mu{}_\nu - \partial^\mu \frac{1}{\Box}\partial_\nu$ being the projector on the transverse vector states. The elements of the matrix operator (4.9) are defined according to (4.8). They include the expressions (4.3) and also

$$
\begin{aligned}
L^{\alpha\beta} &= -i\eta\bar{\psi}\gamma_5\gamma^\alpha\gamma^\beta\,, & M^\beta &= \eta^2\bar{\psi}\gamma^\beta\gamma^\alpha S_\alpha + \eta m\bar{\psi}\gamma_5\gamma^\beta\,, \\
A^\alpha &= 2i\eta\frac{m}{M}\bar{\psi}\gamma_5\gamma^\alpha\,, & B &= 2\eta^2\frac{m}{M}\bar{\psi}\gamma^\beta S_\beta - 2\eta\frac{m^2}{M}\bar{\psi}\gamma_5\,, \\
N &= 4\eta^2\frac{m}{M^2}\bar{\psi}\psi\,, & P^\beta &= -2\eta\gamma^\beta\gamma_5\psi\,, \qquad Q = -4\eta\frac{m}{M}\gamma_5\psi\,.
\end{aligned} \tag{4.10}
$$

The operator $\hat{\mathbf{H}}$ given above might look like the minimal second order operator $(\Box + 2h^\lambda\nabla_\lambda + \Pi)$; but, in fact, it is not minimal because of the projectors $\theta^{\mu\nu}$ in the axial vector-axial vector $t^\perp_\mu$-$t^\perp_\nu$ sector. That is why one cannot directly apply the standard Schwinger-DeWitt expansion to derive the divergent contributions to the one-loop effective action, and some more sophisticated technique is needed.

Let us perform the expansion in the transverse axial vector space, and then apply the generalized Schwinger-DeWitt technique developed by Barvinsky and Vilkovisky [4]. To some extent, the transformations which we are going to do are similar to the ones which have been used for the calculations in high-derivative gravity coupled to matter [19] (see also [5]). Notice that, in the present case, these transformations enable one to perform the calculations in the abelian vector theory. For the massless case, the results are indeed the same as the ones derived with the use of the Faddeev-Popov method.

Since we are dealing with the mixed operator including the boson and fermion sectors, the trace of all products should be understood as a supertrace ($Str$), which implies a positive sign for the bosonic sector and negative sign for the fermionic sector. One can perform the following expansion:

$$
\Gamma^{(1)} = \frac{i}{2}\,\mathrm{Str}\ln\mathbf{H} = \frac{i}{2}\,\mathrm{Str}\ln\begin{pmatrix} \theta\Box & 0 & 0 \\ 0 & \Box & 0 \\ 0 & 0 & \hat{1}\Box \end{pmatrix} - \frac{i}{2}\,\mathrm{Str}\left\{\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left(\hat{\Pi}\frac{1}{\Box}\right)^n\right\}, \tag{4.11}
$$

where the operator $\hat{\Pi}$ corresponds to (4.9):

$$
\hat{\Pi}\frac{1}{\Box} = \left\{\begin{matrix} \theta^{\mu\nu}M^2\frac{1}{\Box} & 0 & \theta^\mu{}_\beta(L^{\beta\alpha}\partial_\alpha + M^\beta)\frac{1}{\Box} \\ 0 & N\frac{1}{\Box} & A^\alpha\partial_\alpha\frac{1}{\Box} + B\frac{1}{\Box} \\ P^\beta\theta_\beta{}^\nu\frac{1}{\Box} & Q\frac{1}{\Box} & (R^\lambda\partial_\lambda + \Pi)\frac{1}{\Box} \end{matrix}\right\}. \tag{4.12}
$$

We are going to use the universal traces of [4], and since we are working in flat space-time, the only non-zero traces, for any given $n$, are

$$
\mathrm{Tr}\left(\partial_{\mu_1}\ldots\partial_{\mu_{2n-4}}\frac{1}{\Box^n}\right)\Bigg|_{\mathrm{div}} = -\frac{2i}{\varepsilon}\int d^4x\,\frac{g^{(n-2)}_{\mu_1\ldots\mu_{2n-4}}}{2^{n-2}(n-1)!}\,. \tag{4.13}
$$



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Here, the standard notational conventions of [4] are used:

$$g^{(0)} = 1\,, \qquad g^{(2)}_{\mu\nu} = g_{\mu\nu}\,, \qquad g^{(4)}_{\mu\nu\alpha\beta} = g_{\mu\nu}g_{\alpha\beta} + g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}\,, \ldots \text{ etc.}$$

It is easy to see, by counting the number of derivative in the terms of the series (4.11), that the divergences appear only for $n = 2, 3, 4$ and that the ones coming from $n = 4$ are completely defined by the fermionic operator (4.1), which we have already taken into account. Therefore, now we only need to work with the terms with $n = 2, 3$.

Consider the $n = 2$ term,

$$\left(\hat{\Pi}\frac{1}{\Box}\right)^2 = \begin{pmatrix} D^{(2)}_1 & E^{(2)}_1 & E^{(2)}_2 \\ E^{(2)}_3 & D^{(2)}_2 & E^{(2)}_4 \\ E^{(2)}_5 & E^{(2)}_6 & D^{(2)}_3 \end{pmatrix}, \tag{4.14}$$

where

$$D^{(2)}_1 = \theta^{\mu\nu}M^4\frac{1}{\Box^2} + \theta^{\mu}{}_{\beta}L^{\beta\alpha}\partial_\alpha\frac{1}{\Box}P^\gamma\theta_\gamma{}^\nu\frac{1}{\Box} + \theta^\mu{}_\beta M^\beta\frac{1}{\Box}P^\gamma\theta_\gamma{}^\nu\frac{1}{\Box}\,, \tag{4.15}$$

$$E^{(2)}_1 = \theta^\mu{}_\beta L^{\beta\alpha}\partial_\alpha\frac{1}{\Box}Q\frac{1}{\Box} + \theta^\mu{}_\beta M^\beta\frac{1}{\Box}Q\frac{1}{\Box}\,, \tag{4.16}$$

$$\begin{aligned} E^{(2)}_2 = {} & \theta^{\mu\rho}M^2\frac{1}{\Box}\theta_{\rho\beta}L^{\beta\alpha}\partial_\alpha\frac{1}{\Box} + \theta^{\mu\rho}M^2\frac{1}{\Box}\theta_{\rho\beta}M^\beta\frac{1}{\Box} + \theta^\mu{}_\beta L^{\beta\alpha}\partial_\alpha\frac{1}{\Box}R^\lambda\partial_\lambda\frac{1}{\Box} + \\ & + \theta^\mu{}_\beta L^{\beta\alpha}\partial_\alpha\frac{1}{\Box}\Pi\frac{1}{\Box} + \theta^\mu{}_\beta M^\beta\frac{1}{\Box}R^\lambda\partial_\lambda\frac{1}{\Box} + \theta^\mu{}_\beta M^\beta\frac{1}{\Box}\Pi\frac{1}{\Box}\,, \end{aligned} \tag{4.17}$$

$$E^{(2)}_3 = A^\alpha\partial_\alpha\frac{1}{\Box}P^\beta\theta_\beta{}^\nu\frac{1}{\Box} + B\frac{1}{\Box}P^\beta\theta_\beta{}^\nu\frac{1}{\Box}\,, \tag{4.18}$$

$$D^{(2)}_2 = N\frac{1}{\Box}N\frac{1}{\Box} + A^\alpha\partial_\alpha\frac{1}{\Box}Q\frac{1}{\Box} + B\frac{1}{\Box}Q\frac{1}{\Box}\,, \tag{4.19}$$

$$\begin{aligned} E^{(2)}_4 = {} & N\frac{1}{\Box}A^\alpha\partial_\alpha\frac{1}{\Box} + N\frac{1}{\Box}B\frac{1}{\Box} + A^\alpha\partial_\alpha\frac{1}{\Box}R^\lambda\partial_\lambda\frac{1}{\Box} + A^\alpha\partial_\alpha\frac{1}{\Box}\Pi\frac{1}{\Box} + \\ & + B\frac{1}{\Box}R^\lambda\partial_\lambda\frac{1}{\Box} + B\frac{1}{\Box}\Pi\frac{1}{\Box}\,, \end{aligned} \tag{4.20}$$

$$E^{(2)}_5 = P^\beta\theta_\beta{}^\rho\frac{1}{\Box}\theta_\rho{}^\nu M^2\frac{1}{\Box} + R^\lambda\partial_\lambda\frac{1}{\Box}P^\beta\theta_\beta{}^\nu\frac{1}{\Box} + \Pi\frac{1}{\Box}P^\beta\theta_\beta{}^\nu\frac{1}{\Box}\,, \tag{4.21}$$

$$E^{(2)}_6 = Q\frac{1}{\Box}N\frac{1}{\Box} + R^\lambda\partial_\lambda\frac{1}{\Box}Q\frac{1}{\Box} + \Pi\frac{1}{\Box}Q\frac{1}{\Box}\,, \tag{4.22}$$

$$\begin{aligned} D^{(2)}_3 = {} & P^\gamma\theta_\gamma{}^\rho\frac{1}{\Box}\theta_{\rho\beta}L^{\beta\alpha}\partial_\alpha\frac{1}{\Box} + P^\gamma\theta_\gamma{}^\rho\frac{1}{\Box}\theta_{\rho\beta}M^\beta\frac{1}{\Box} + Q\frac{1}{\Box}A^\alpha\partial_\alpha\frac{1}{\Box} + \\ & + Q\frac{1}{\Box}B\frac{1}{\Box} + \left\{R^\lambda\partial_\lambda\frac{1}{\Box} + \Pi\frac{1}{\Box}\right\}^2. \end{aligned} \tag{4.23}$$

Here, we use $\mathrm{Tr}(AB) = \pm\,\mathrm{Tr}(BA)$ for the operators, depending on their Grassmann parity. Disregarding the purely fermionic contributions (which we already calculated), we obtain for the divergent part:

$$\begin{aligned} -\frac{1}{2}\,\mathrm{Str}\left(\hat{\Pi}\frac{1}{\Box}\right)^2 = -\frac{1}{2}\,\mathrm{Tr}\Big\{ & \theta^{\mu\nu}M^4\frac{1}{\Box^2} + 2\theta^\mu{}_\beta L^{\beta\alpha}\partial_\alpha\frac{1}{\Box}P^\gamma\theta_\gamma{}^\nu\frac{1}{\Box} + 2\theta^\mu{}_\beta M^\beta\frac{1}{\Box}P^\gamma\theta_\gamma{}^\nu\frac{1}{\Box} + \\ & + N^2\frac{1}{\Box^2} + 2A^\alpha\partial_\alpha\frac{1}{\Box}Q\frac{1}{\Box} + 2B\frac{1}{\Box}Q\frac{1}{\Box}\Big\}\,. \end{aligned} \tag{4.24}$$



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After some involved commutations (which we do not discuss because they are in fact similar to the ones described in [19, 5]), we arrive at

$$
\begin{aligned}
-\frac{1}{2}\,\mathrm{Str}\left(\hat{\Pi}\frac{1}{\Box}\right)^2 = -\frac{1}{2}\,\mathrm{Tr}\Big\{&\theta^{\mu\nu}M^4\frac{1}{\Box^2} - 4L^{\gamma\alpha}(\partial^\rho P_\gamma)\partial_\alpha\partial_\rho\frac{1}{\Box^3} + 2L^{\gamma\alpha}(\partial_\alpha P_\gamma)\frac{1}{\Box^2}+\\
&+ 4L^{\beta\alpha}(\partial^\rho P^\gamma)\partial_\alpha\partial_\beta\partial_\gamma\partial_\rho\frac{1}{\Box^4} - 2L^{\beta\alpha}(\partial_\alpha P^\gamma)\partial_\beta\partial_\gamma\frac{1}{\Box^3} +\\
&+ 2M^\gamma P_\gamma\frac{1}{\Box^2} - 2M^\beta P^\gamma\partial_\beta\partial_\gamma\frac{1}{\Box^3} + N^2\frac{1}{\Box^2} -\\
&- 4A^\alpha(\partial^\rho Q)\partial_\alpha\partial_\rho\frac{1}{\Box^3} + 2A^\alpha(\partial_\alpha Q)\frac{1}{\Box^2} + 2BQ\frac{1}{\Box^2}\Big\}
\end{aligned}
\tag{4.25}
$$

Using (4.13), we get

$$
\begin{aligned}
-\frac{1}{2}\,\mathrm{Str}\left(\hat{\Pi}\frac{1}{\Box}\right)^2\Big|_{\mathrm{div}} = \frac{i}{\varepsilon}\int d^4x\,\Big\{&3M^4 + \frac{2}{3}L^{\gamma\alpha}(\partial_\alpha P_\gamma) + \frac{1}{6}L_\alpha{}^\alpha(\partial_\gamma P^\gamma) + \frac{1}{6}L^{\gamma\alpha}(\partial_\gamma P_\alpha)+\\
&+ \frac{3}{2}M^\gamma P_\gamma + A^\alpha(\partial_\alpha Q) + 2BQ + N^2\Big\}.
\end{aligned}
\tag{4.26}
$$

By direct substitution of the expressions $A^\alpha, \ldots, \Pi$, and after some algebra, we arrive at the partial result:

$$
\begin{aligned}
-\frac{1}{2}\,\mathrm{Str}\left(\hat{\Pi}\frac{1}{\Box}\right)^2\Big|_{\mathrm{div}} = \frac{i}{\varepsilon}\int d^4x\,\Big\{&3M^4 + 16\eta^3\frac{m^2}{M^2}\bar{\psi}\gamma_5\not{S}\psi + \left(16\eta^2\frac{m^3}{M^2} - 12\eta^2 m\right)\bar{\psi}\psi-\\
&- 6\eta^3\bar{\psi}\gamma_5\not{S}\psi + 8i\eta^2\frac{m^2}{M^2}\bar{\psi}\not{\partial}\psi + 16\eta^4\frac{m^2}{M^4}(\bar{\psi}\psi)^2\Big\}.
\end{aligned}
\tag{4.27}
$$

Note that each term inside this integral has the dimension of $[mass]^4$, despite the unusual form of the contribution of the torsion mass $M$.

Consider the $n = 3$ term. Again, omitting all the contributions into the fermionic sector, we write only the relevant terms:

$$
\frac{1}{3}\,\mathrm{Str}\left(\hat{\Pi}\frac{1}{\Box}\right)^3\Big|_{\mathrm{div}} = \frac{1}{3}\,\mathrm{Tr}\left(3\theta_{\mu\rho}L_\rho{}^\alpha\partial_\alpha\frac{1}{\Box}R^\lambda\partial_\lambda\frac{1}{\Box}P^\beta\theta_\beta{}^\nu\frac{1}{\Box} + 3A^\alpha\partial_\alpha\frac{1}{\Box}R^\lambda\partial_\lambda\frac{1}{\Box}Q\frac{1}{\Box}\right).
\tag{4.28}
$$

After some algebra, and using the universal traces (4.13), we obtain

$$
\begin{aligned}
\frac{1}{3}\,\mathrm{Str}\left(\hat{\Pi}\frac{1}{\Box}\right)^3\Big|_{\mathrm{div}} = -\frac{2i}{3\varepsilon}\int d^4x\,\Big\{&\frac{5}{8}L^{\rho\alpha}R_\alpha P_\rho - \frac{1}{8}L^\alpha{}_\alpha R_\lambda P^\lambda-\\
&- \frac{1}{8}L^{\rho\alpha}R_\rho P_\alpha + \frac{3}{4}A_\alpha R^\alpha Q\Big\}.
\end{aligned}
\tag{4.29}
$$

The relevant terms in the partial result ($n = 3$) are

$$
\frac{1}{3}\,\mathrm{Str}\left(\hat{\Pi}\frac{1}{\Box}\right)^3\Big|_{\mathrm{div}} = \frac{i}{\varepsilon}\int d^4x\,\left\{6\eta^3\bar{\psi}\gamma_5\not{S}\psi - 24\eta^3\frac{m^2}{M^2}\bar{\psi}\gamma_5\not{S}\psi\right\}.
\tag{4.30}
$$

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Summing up the contributions to the one-loop divergences of (4.11), coming from (4.27) and (4.30) and (4.6), we obtain the complete expression for the one-loop divergences:

$$\Gamma^{(1)}_{\rm div} = -\frac{\mu^{n-4}}{\varepsilon}\int d^nx\left\{-\frac{2\eta^2}{3}S_{\mu\nu}S^{\mu\nu} + 8m^2\eta^2S^\mu S_\mu - 2m^4 + \frac{3}{2}M^4 + \right. \qquad (4.31)$$

$$\left. + \left(8\eta^2\frac{m^3}{M^2} - 6\eta^2 m\right)\bar{\psi}\psi + 8\eta^4\frac{m^2}{M^4}(\bar{\psi}\psi)^2 + 4i\eta^2\frac{m^2}{M^2}\bar{\psi}\gamma^\mu D^*_\mu\psi\right\}.$$

It is interesting to notice that the above expression (4.31) is not gauge invariant. It is not difficult to see that the non-invariant terms come as a contribution of the scalar $\varphi$. This indicates that, unlike the massive (abelian) vector field (see appendix A), for the massive axial vector the violation of the symmetry (2.5) is not soft. Therefore, we have confirmed our previous analysis based on the Ward-Takahashi identities. More detailed consideration of the renormalization is presented in the next section.

## 5. Renormalization and renormalization group

The expression (4.31) for the 1-loop divergences in the theory (2.6) has two non-invariant pieces. The first one comes from the $\bar{\psi}\gamma^\mu D^*_\mu\psi$ term, which is not invariant with respect to (2.5). In fact, this divergence produces just a slight change in the renormalization of the coupling constant $\eta$, so that the softly broken symmetry (2.5) can be maintained at the quantum level. The second term is essentially non-invariant $(\bar{\psi}\psi)^2$-structure. The renormalizability of the theory requires the $(\bar{\psi}\psi)^2$ term to be introduced into the classical action, so that the corresponding counterterm can be removed by means of the renormalization of the corresponding parameter. Indeed, one can calculate again the 1-loop divergences taking this term into account. On the other hand, this is not necessary, because there is no one-loop diagram containing this $(\bar{\psi}\psi)^2$-vertex which could contribute to the dangerous longitudinal divergence.[3] Unfortunately, such a diagram exists at the two-loop level. We postpone the analysis of the two-loop diagrams to the next section, and consider now, in some details, the one-loop renormalization.

The appearance of the new $(\bar{\psi}\psi)^2$-vertex shows that the fermion-torsion theory cannot be consistent even as an effective quantum field theory, at least without some additional restrictions being imposed. Let us try to introduce some additional restrictions on the value of the torsion mass, $M$. Suppose $m \ll M$. This means that the torsion mass is much (let us say, some orders) larger than the mass of any fermion interacting with torsion. Alternatively, one can suppose that torsion interacts only with massless spinors (this case is free from any problem at the quantum level,

[3]Let us remind the danger of the $(\partial_\alpha S^\alpha)^2$-type counterterm, which spoils both the renormalizability and the unitarity of the theory.



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but the existence of the massless spinors in the SM is nowadays problematic) or very light fermions and decouples, by definition, from heavy fermions. Since our simplified consideration does not distinguish heavy and light quarks, leptons etc, we just accept $m \ll M$ for a moment. Then, both types of non-invariant counterterms carry very small coefficients, proportional to $(m/M)^2$. Suppose we include the "dangerous" interaction $(\bar{\psi} \cdot \psi)^2$ into the action, but with a very small coupling of the order $\lambda \sim m^2/M^4$. This relation will not be violated by the renormalization group running of the coupling $\lambda$, and hence the renormalizability is achieved with a very weak coupling $(\bar{\psi}\psi)^2$. As we shall see in the next section, the two-loop contribution to the dangerous longitudinal counterterm $(\partial_\alpha S^\alpha)^2$ contains the $(\bar{\psi}\psi)^2$-vertex. Therefore, one finds it possible to preserve renormalizability if the $(\partial_\alpha S^\alpha)^2$-term is included into the action (2.6) with a coefficient $b \sim (m/M)^4$.

Formally, if the $(\partial_\alpha S^\alpha)^2$-structure is present, unitarity is broken, since the corresponding degree of freedom is a ghost. This term, along with the canonical kinetic term $S_{\mu\nu}S^{\mu\nu}$, will unavoidably plague the spectrum with unphysical modes: either a tachyonic or a negative-norm state (ghost) excitation will show up as a spin-1 or a scalar excitation. However, unitarity is still ensured in the spinor sector of the theory; it may break only in the *torsion-torsion* sector. Let us consider some low-energy amplitude involving *in*-states of the propagating transverse torsion. In order to generate *out*-states of the longitudinal torsion, one has to consider the diagrams with corresponding vertices. Such vertices are absent at tree-level, and the ones, which involve a non-invariant $(\bar{\psi} \cdot \psi)^2$-interaction show up, as we shall see in the next section, at the second loop only. Then, the longitudinal *out*-state is suppressed by the coefficient $(m/M)^4$. Therefore, in the low-energy amplitudes of the torsion (axial vector $S_\mu$) scattering, the unitarity is maintained with the precision $(m/M)^4$.

Consider the one-loop renormalization and the corresponding renormalization group in some details. The relations between bare and renormalized fields and the coupling $\eta$ follow from (4.31):

$$
\begin{aligned}
S_\mu^{(0)} &= \mu^{\frac{n-4}{2}} S_\mu \left(1 + \frac{1}{\epsilon} \cdot \frac{4\eta^2}{3}\right), \qquad \psi^{(0)} = \mu^{\frac{n-4}{2}} \psi \left(1 + \frac{1}{\epsilon} \cdot \frac{2\eta^2 m^2}{M^2}\right), \\
\eta^{(0)} &= \mu^{\frac{4-n}{2}} \left(\eta - \frac{1}{\epsilon} \cdot \frac{4\eta^3}{3} \cdot \left[1 + 6\frac{m^2}{M^2}\right]\right). \qquad (5.1)
\end{aligned}
$$

Similar relations for the parameter $\tilde{\lambda} = \frac{M^4}{m^2}\lambda$ of the $\lambda(\bar{\psi}\psi)^2$- interaction, have the form:

$$
\tilde{\lambda}^{(0)} = \mu^{4-n} \left\{ \tilde{\lambda} + \frac{8\eta^4}{\varepsilon} + \frac{16\eta^2}{\varepsilon} \frac{m^2}{M^2} \tilde{\lambda} + \frac{20\eta^2}{3\varepsilon} \tilde{\lambda} \right\}. \qquad (5.2)
$$

These relations lead to a renormalization group equation for $\eta$, which contains a new term proportional to $(m/M)^2$:

$$
(4\pi)^2 \frac{d\eta^2}{dt} = \frac{8}{3} \left[1 + 6\frac{m^2}{M^2}\right] \eta^4, \qquad \eta(0) = \eta_0. \qquad (5.3)
$$



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Indeed, for the case $m \ll M$ and in the low-energy region, this equation reduces to the one presented in [2] (that is identical to the similar equation of QED). In any other case, the theory of torsion coupled to the massive spinors is inconsistent, and equation (5.3) is meaningless.

One can also write down the renormalization group equation for the parameter $\tilde{\lambda}$ defined above. Using (5.2), we arrive at the following equation:

$$(4\pi)^2 \frac{d\tilde{\lambda}}{dt} = 8\eta^4\,. \tag{5.4}$$

This equation confirms the lack of a too fast running for this parameter. Indeed, all the last consideration is valid only under the assumption that $m \ll M$ and has very restricted sense.

## 6. Two-loop diagrams

Let us investigate the 2-loop diagrams contributing to the propagator of the axial vector, $S_\mu$. The question we intend to answer is whether there are longitudinal divergences at the two-loop level. Therefore, it is reasonable to start from the diagrams which can exhibit $1/\epsilon^2$-divergences,[4] and only if none of them are found, we explore the $1/\epsilon$-pole, which is always more complicated to calculate.

The leading $1/\epsilon^2$-two-loop divergences of the mass operator for the axial vector $S_\mu$ come from two distinct types of diagrams: the ones with the $(\bar{\psi}\psi)^2$-vertex and the ones without this vertex. As we shall ensure, the most dangerous diagrams are those with 4-fermion interaction. As we have seen in the last two sections, this kind of interaction is a remarkable feature of the axial vector theory, which is absent in a massive vector theory. Now, we shall calculate divergent $1/\epsilon^2$-contributions from two diagrams with the $(\bar{\psi}\psi)^2$-vertex, using the expansion suggested in [2]; later on, in appendix B, this calculation will be checked using Feynman parameters.

Consider first the diagram of figure 1. This graph can be expressed, after making some commutations of the $\gamma$-matrices, as

$$\Pi^1_{\mu\nu} = -\lambda\eta^2 \,\mathrm{tr}\,\{I_\nu \cdot I_\mu\}\,, \tag{6.1}$$

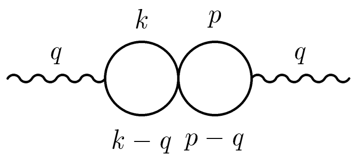


**Figure 1**

where $\lambda \sim \frac{m^2}{M^4}$ is the coupling of the four-fermion vertex, the trace is taken over the Dirac spinor space and

$$I_\nu(p) = \int \frac{d^dp}{(2\pi)^d} \frac{\not{p} - m}{p^2 - m^2} \gamma_\nu \gamma_5 \frac{\not{p} - \not{q} - m}{(p-q)^2 - m^2}\,. \tag{6.2}$$

Following [2], we can perform the expansion

$$\frac{1}{(p-q)^2 - m^2} = \frac{1}{p^2 - m^2} \sum_{n=0}^{\infty} (-1)^n \left( \frac{-2pq + q^2}{p^2 - m^2} \right)^n. \tag{6.3}$$

[4] In this paper, we adopt the dimensional regularization. All necessary integrals may be found in [20, 21].

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Now, as far as we are working within an effective field theory framework, it is possible to omit the powers of $q$ higher than 2. These terms can give contributions to the divergences, but only to the ones with higher derivatives, and they are, therefore, out of our interest.

When performing the integrations, we trace just the divergent parts, thus arriving (using the integrals from [21]) at the expressions:

$$I_\nu = \frac{i}{\epsilon}\left\{-\frac{1}{6}q^2\gamma_\nu - 2m^2\gamma_\nu - \frac{1}{6}\gamma_\alpha\gamma_\nu\gamma_\beta q^\alpha q^\beta + mq_\nu\right\} + \cdots, \tag{6.4}$$

where the dots stand for the finite and higher-derivative divergent terms. Substituting this into (6.1), we obtain the leading divergence of the diagram of figure 1:

$$\Pi^{1,div}_{\mu\nu} = -\frac{\lambda\eta^2}{\epsilon^2}\left\{+16m^4\eta_{\mu\nu} + \frac{28}{3}m^2 q_\mu q_\nu - \frac{16}{3}m^2 q^2\eta_{\mu\nu}\right\} + \cdots \tag{6.5}$$

This result shows that the construction of the first diagram contains an $1/\epsilon^2$-longitudinal counterterm.

Consider the second two-loop diagram depicted in figure 2. Its contribution to the polarization operator, $\Pi^2_{\mu\nu}$, is written, after certain transformations, in the following way:

$$\Pi^2_{\mu\nu} = -\lambda\eta^2\; tr\left\{I_{\nu\mu}\cdot J\right\}, \tag{6.6}$$

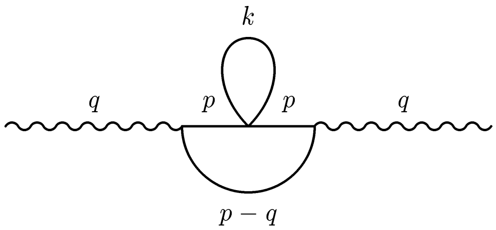

**Figure 2**

where

$$I_{\nu\mu} = \int \frac{d^dp}{(2\pi)^d}\frac{\not p - m}{p^2 - m^2}\gamma_\nu \frac{\not p - \not q + m}{(p-q)^2 - m^2}\gamma_\mu \frac{\not p - m}{p^2 - m^2} \tag{6.7}$$

and

$$J = \int \frac{d^dp}{(2\pi)^d}\frac{\not k - m}{k^2 - m^2}\,. \tag{6.8}$$

It proves useful to introduce the following $\gamma$-matrix definitions:

$$\begin{aligned}
A_{\alpha\nu\beta\mu\rho} &= \gamma_\alpha\gamma_\nu\gamma_\beta\gamma_\mu\gamma_\rho\,,\\
B_{\alpha\nu\mu\beta} &= -q^\rho\gamma_\alpha\gamma_\nu\gamma_\rho\gamma_\mu\gamma_\beta + m(\gamma_\alpha\gamma_\nu\gamma_\mu\gamma_\beta - \gamma_\nu\gamma_\alpha\gamma_\mu\gamma_\beta - \gamma_\alpha\gamma_\nu\gamma_\beta\gamma_\mu)\,,\\
C_{\alpha\nu\mu} &= m^2(\gamma_\nu\gamma_\alpha\gamma_\mu - \gamma_\nu\gamma_\mu\gamma_\alpha - \gamma_\nu\gamma_\alpha\gamma_\mu) + mq^\beta(\gamma_\nu\gamma_\beta\gamma_\mu\gamma_\alpha + \gamma_\alpha\gamma_\nu\gamma_\beta\gamma_\mu)\,,\\
D_{\nu\mu} &= -m^2q^\beta\gamma_\nu\gamma_\beta\gamma_\mu + m^3\gamma_\nu\gamma_\mu\,.
\end{aligned}$$

Then, the first integral can be written as

$$I_{\nu\mu} = \int \frac{d^dp}{(2\pi)^d}\frac{A_{\alpha\nu\beta\mu\rho}\cdot p^\alpha p^\beta p^\rho + B_{\alpha\nu\mu\beta}\cdot p^\alpha p^\beta + C_{\alpha\nu\mu}p^\alpha + D_{\nu\mu}}{(p^2-m^2)^2\,((p-q)^2 - m^2)}\,. \tag{6.9}$$



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Using the expansion (6.3), and disregarding higher powers of $q$, as well as odd powers of $p$ in the numerator of the resulting integral, one obtains, after using standard results [21]:

$$I_{\nu\mu} = \frac{i}{\epsilon}\left\{\frac{1}{4}B^{\alpha}{}_{\nu\mu\alpha} + \frac{1}{12}(A^{\alpha}{}_{\nu\alpha\mu\rho} + A^{\alpha}{}_{\nu\rho\mu\alpha} + A_{\rho\nu\alpha\mu}{}^{\alpha})q^{\rho}\right\} + \cdots \tag{6.10}$$

which gives, after some algebra,

$$I_{\nu\mu} = \frac{i}{\epsilon}\left\{m\gamma_{\mu}\gamma_{\nu} + 3m\eta_{\mu\nu} - \frac{2}{3}\gamma_{\rho}q^{\rho}\eta_{\mu\nu} + \frac{1}{3}\gamma_{\mu}q_{\nu} + \frac{1}{3}\gamma_{\nu}q_{\mu}\right\} + \cdots \tag{6.11}$$

The divergent contribution to $J$ is

$$J = -\frac{i}{\epsilon}m^3 + \cdots \tag{6.12}$$

Now, the calculation of (6.6) is straightforward:

$$\Pi_{\mu\nu} = \frac{\lambda\eta^2}{\epsilon^2}8m^4\eta_{\mu\nu} + \cdots \tag{6.13}$$

As we see, this diagram does not contribute to the kinetic counterterm (with accuracy of the higher-derivative terms), and hence the cancellation of the contributions to the longitudinal counterterm coming from $\Pi^1_{\mu\nu}$ do not take place. This result is reproduced in the appendix B, with the help of the Feynman parameters.

One has to notice that other two-loop diagrams do not include the $(\bar{\psi}\psi)^2$-vertex. Thus, even if those diagrams contribute to the longitudinal counterterm, the cancellation with $\Pi^1_{\mu\nu}$ should require some special fine-tuning between $\lambda$ and $\eta$. In fact, one can prove, without explicit calculation, that the remaining two-loop diagrams of figures 3 and 4 do not contribute to the longitudinal $1/\epsilon^2$-pole. In order to see this, let us notice that the leading (in our case $1/\epsilon^2$) divergence may be obtained by consequent substitution of the contributions from the subdiagrams by their local divergent components. Since the local counterterms produced by the subdiagrams of the two-loop graphs depicted in figures 3 and 4 are minus the one-loop expression (4.31), the corresponding divergent vertices are $1/\epsilon$ factor classical vertices. Hence, in the leading $1/\epsilon^2$-divergences of the diagrams of figures 3 and 4, one meets again the same expressions as in (4.31). The result of our consideration is, therefore, the non-cancellation of the $1/\epsilon^2$-longitudinal divergence (6.5). This means that the theory (2.6), without additional restrictions on the torsion mass, like $m \ll M$, is inconsistent at the quantum level.

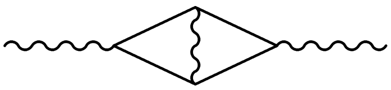

**Figure 3**

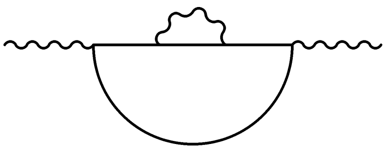

**Figure 4**

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## 7. Conclusions

We have investigated, in more details than in the previous works [2], the quantum field theory of the fermion-torsion system. The torsion is presented by its purely antisymmetric part, equivalent to the axial vector $S_\mu$. It was shown that renormalizability and unitarity may be achieved only in the case of massless spinors coupled to massless torsion, without scalar fields. According to recent data on the neutrino oscillations, all existing fermions have a non-zero mass. Probably, this means that they also interact with the Higgs scalar. Thus, it is clear that torsion cannot be implemented in a Standard Model scenario or, at least, into its versions which are available to the date.

Alternatively, one has to input very severe restrictions on the torsion mass, which has to be much greater than the mass of the heaviest fermion (say, t-quark, with a mass of 175 GeV), and use an effective quantum field theory approach, restricting considerations to the low-energy amplitudes only. This approach implies the existence of a fundamental theory which is valid at higher energies. The effective theory may be used only at the energies essentially smaller than the typical mass scale of the fundamental theory. If the mass of torsion is comparable to this fundamental scale, all the torsion degrees of freedom may be described directly in the framework of the fundamental theory.

Hence, in order to have propagating torsion, one has to satisfy a double inequality:

$$m_{\text{fermion}} \ll M_{\text{torsion}} \ll M_{\text{fundamental}}\,. \tag{7.1}$$

Usually, the fundamental scale is associated with the Planck mass, $M_{Pl} \approx 10^{19}$ GeV,[5] and therefore we still have a huge gap on the energy spectrum, which is not completely covered by the present theoretical consideration. Of course, this gap cannot be closed by any experiment, because the mass of torsion is too big. Even the restrictions coming from the contact experiments [2] achieve only the region $M < 3\,\text{TeV}$, and that is not enough to satisfy (7.1) for all the fermions of the Standard Model. It is clear that the existence of a torsion-interacting fermions with mass of many orders larger than $m_t$ (like the ones which are expected in many GUT's) can close the gap on the particle spectrum and "forbid" an independent torsion.

The situation with torsion is similar to the one with quantum gravity. In both cases, there is a conflict between renormalizability (which lacks, in case of gravity, for the Einstein theory) and unitarity which is violated in high-derivative models [15, 22]. In some sense, this analogy is natural, because both the metric and torsion represent the internal aspects of the space-time manifold rather than usual fields. Therefore, one of the options is to give up the quantization of these two fields and consider

[5]As a by product, our study shows that if the real fundamental scale is just a few orders above TeV, there is no room for an independent propagating torsion. Thus, one cannot incorporate torsion into the recent discussion of the TeV-gravity (see, for instance, [24]).



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them only as a classical background. If one does not accept this option, it is possible to consider both the metric and torsion as effective low-energy interactions resulting from a more fundamental theory like string. Both the metric and torsion result from string, but the crucial difference is that metric has massless degrees of freedom while torsion appears to have a mass in all known versions of string theory [23]. It is interesting that the study of an effective quantum field theory for the metric does not meet major difficulties [25, 1], while the consistency of the theory requires a lower-bound (7.1) on the torsion mass. One can guess that this is more than an accidental coincidence.

Indeed, it is possible that some new symmetries will be discovered, which make the consistent quantum theory of the propagating torsion possible. However, in the framework of the well-established results, the most natural supposition is perhaps that the torsion does not exist as an independent field, or that it is purely classical field which should not be quantized.

On the other hand, our study does not close the possibility of having composite torsion, which can appear, for instance, as a vacuum condensate of the light (or maybe even massless) spinor fields. This possibility deserves, from our point of view, special investigation.

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## A. Calculation of divergences for massive vector, massless vector and axial vector coupled to fermions

All the calculations below shall be performed on a flat background. Consider first the theory for massive vectors with the action (3.1)

$$S = \int d^4x \left\{ -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} M^2 V_\mu V^\mu + i\bar{\psi}(\gamma^\mu D_\mu - im)\psi \right\}, \qquad \text{(A.1)}$$

where $D_\mu = \partial_\mu - igV_\mu$ and $V_{\mu\nu} = \partial_\nu V_\mu - \partial_\mu V_\nu$. In the framework of the background field method, one performs the shift

$$\psi \to \psi' = e^{i\frac{g}{M}\varphi}(\psi + \eta)\,,$$

$$\bar{\psi} \to \bar{\psi}' = (\bar{\psi} + \bar{\eta}) e^{-i\frac{g}{M}\varphi} ,$$
$$V_\mu \to V_\mu' = V_\mu + t_\mu^\perp + \frac{1}{M}\partial_\mu \varphi .$$

As the scalar field does not couple to any other field, the $\varphi$-sector can be successfully factored out. The quadratic (in the quantum fields $t_\mu^\perp, \eta, \bar{\psi}$) part of the action is, after the change of the variables $\eta = -\frac{i}{2}(\gamma^\mu D_\mu + im)\tau$, written in the form

$$S^{(2)} = \frac{1}{2} \int d^4x \, (\, t_\mu^\perp \quad \bar{\eta} \,) \, \hat{\mathbf{H}} \begin{pmatrix} t_\nu^\perp \\ \tau \end{pmatrix} , \tag{A.2}$$

where

$$\hat{\mathbf{H}} = \left\{ \begin{array}{cc} \theta^{\mu\nu}(\Box + M^2) & \theta^\mu{}_\beta (L^{\beta\alpha}\partial_\alpha + N^\beta) \\ h^\beta \theta_\beta{}^\nu & \Box + R^\lambda \partial_\lambda + \Pi \end{array} \right\} , \tag{A.3}$$

and

$$\begin{aligned} L^{\beta\alpha} &= -ig\bar{\psi}\gamma^\beta\gamma^\alpha , \qquad & N^\beta &= -g^2\bar{\psi}\gamma^\beta\gamma^\alpha V_\alpha + mg\bar{\psi}\gamma^\beta , \\ h^\beta &= 2g\gamma^\beta\psi , & R^\lambda &= -2igV^\lambda , \\ \Pi &= \frac{i}{2}g\gamma^\mu\gamma^\nu V_{\mu\nu} - ig(\partial_\mu V^\mu) - g^2 V^\mu V_\mu + m^2 . \end{aligned}$$

The operator (A.3) is simpler than the one in (4.9), because of the decoupling of the scalar mode in the vector case.

Now, we can evaluate the one-loop divergences of the effective action in the theory (3.1). For this, we use the same expansion as in (4.11) but with

$$\hat{\Pi}\frac{1}{\Box} = \left\{ \begin{array}{cc} \theta^{\mu\nu} M^2 \frac{1}{\Box} & \theta^\mu{}_\beta (L^{\beta\alpha}\partial_\alpha + N^\beta)\frac{1}{\Box} \\ h^\beta \theta_\beta{}^\nu \frac{1}{\Box} & (R^\lambda\partial_\lambda + \Pi)\frac{1}{\Box} \end{array} \right\} , \tag{A.4}$$

and, looking for logarithmic divergences, restrict our consideration to the terms with $n = 2, 3, 4$. Also we notice that, as for the axial vector, $n = 4$ contributions are coming from the fermion loop and can be easily derived by standard means [5]. The $n = 2, 3$ terms can be worked out exactly like for the axial vector case, and the partial results for the divergences read

$$-\frac{1}{2}\,\mathrm{Str}\left(\hat{\Pi}\frac{1}{\Box}\right)^2 = \frac{i}{\varepsilon}\int d^4x \left\{ 3M^4 + 6g^3\bar{\psi}\not{V}\psi + 12g^2 m\bar{\psi}\psi \right\} \tag{A.5}$$

and

$$\frac{1}{3}\,\mathrm{Str}\left(\hat{\Pi}\frac{1}{\Box}\right)^3\bigg|_{\mathrm{div}} = \frac{i}{\varepsilon}\int d^4x \left\{ -6g^3\bar{\psi}\not{V}\psi \right\} . \tag{A.6}$$

Adding the standard contribution from the fermion loop, we arrive at the complete expression for the divergences

$$\Gamma^{(1)}_{\mathrm{div}} = -\frac{1}{\varepsilon}\int d^4x \left\{ \frac{3}{2}M^4 - 6g^2 m\bar{\psi}\psi + \frac{2g^2}{3}V_{\mu\nu}V^{\mu\nu} \right\} . \tag{A.7}$$



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In the cases of the massless vector coupled to spinor field (QED), and the massless axial vector coupled to massless spinor, the 1-loop calculation can be done in a standard manner with the help of the Faddeev-Popov method. However, in order to check our calculational method, we performed these calculations in the same way as for the massive cases. The most of the intermediate calculations can be easily restored using the massive cases, so we shall give just a main results.

For the fermion coupled to the massless vector (QED), the calculation is very simple and the divergences have the well-known form:

$$\Gamma^{(1)}_{div} = \frac{1}{\varepsilon} \int d^4x \left\{ -6e^2 m \bar{\psi}\psi + \frac{2e^2}{3} F^2_{\mu\nu} \right\} . \tag{A.8}$$

Consider, in some more details, the theory of the massless axial vector coupled to the massless Dirac spinor. The classical action is

$$S = \int d^4x \left\{ -\frac{1}{4} S_{\mu\nu} S^{\mu\nu} + i\bar{\psi}\gamma^\mu D_\mu \psi \right\} , \tag{A.9}$$

where the covariant derivative is the same as for the massive case. This action is completely invariant under the gauge transformation (2.5). Performing the change of variables and applying the background field method, as described in section 4, we can write the bilinear form of the action in the form

$$\hat{\mathbf{H}} = \begin{pmatrix} \theta^{\mu\nu}(\Box + M^2) & \theta^{\mu}{}_{\beta}(L^{\beta\alpha}\partial_\alpha + M^\beta) \\ P_\beta \theta^{\beta\nu} & \hat{1}\Box + R^\lambda \partial_\lambda + \Pi \end{pmatrix} , \tag{A.10}$$

where

$$L^{\alpha\beta} = -i\eta\bar{\psi}\gamma_5\gamma^\alpha\gamma^\beta , \qquad M^\beta = \eta^2 \bar{\psi}\gamma^\beta\gamma^\alpha S_\alpha , \qquad P^\beta = -2\eta\gamma^\beta\gamma_5\psi . \tag{A.11}$$

The expansion for $\frac{i}{2}\,\mathrm{Tr}\ln\hat{\mathbf{H}}$ and the remaining calculations will produce almost the same intermediate formulas as for the fermion-massive vector calculation. The reason is that the matrices $\hat{\mathbf{H}}$ have many identical structures, the only difference lying on the equations (A.11) above.

We have, after substituting (A.11) into the previous general formulae, noticed that two contributions cancel

$$-\frac{1}{2}\,\mathrm{Str}\left(\hat{\Pi}\frac{1}{\Box}\right)^2 \Bigg|_{\mathrm{div}} = \frac{i}{\varepsilon}\int d^4x \left\{ -6\eta^3 \bar{\psi}\gamma_5 \not{S}\psi \right\} \tag{A.12}$$

$$\frac{1}{3}\,\mathrm{Str}\left(\hat{\Pi}\frac{1}{\Box}\right)^3 \Bigg|_{\mathrm{div}} = \frac{i}{\varepsilon}\int d^4x \left\{ 6\eta^3 \bar{\psi}\gamma_5 \not{S}\psi \right\} , \tag{A.13}$$

and the general expression for the divergences is completely defined by the fermion loop:

$$\Gamma^{(1)}_{div} = \frac{1}{\varepsilon}\int d^4x \frac{2\eta^2}{3} S^2_{\mu\nu} . \tag{A.14}$$

It is indeed gauge invariant. The same divergence follows from the standard calculation using the Faddeev-Popov method.

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## B. Two-loop calculation using Feynman parametrization

Here, we start from the expression (6.2), performing in the denominator the Feynman parametrization:

$$\frac{1}{ab} = \int_0^1 \frac{dx}{\{ax + (1-x)b\}^2} \tag{B.1}$$

Following the standard procedures in dimensional regularization, we have to change the integration variable as $p \to p' = p - qx$. After that, one meets some known integrals and get

$$I_\nu = \frac{\Gamma(\epsilon)}{(4\pi)^2} \int_0^1 dx \left\{ \Delta\gamma_\nu + \Delta^{-\epsilon} D_\nu \right\}, \tag{B.2}$$

where

$$\begin{aligned} \Delta &= q^2 x(1-x) - m^2 \, , \\ D_\nu &= \gamma_\alpha\gamma_\nu\gamma_\beta q^\alpha q^\beta x^2 + x p^\alpha(-2m\eta_{\nu\alpha} + 2m\gamma_\alpha\gamma_\nu - \gamma_\alpha\gamma_\nu\gamma_\beta q^\beta) + \\ &\quad + m q^\alpha \gamma_\nu\gamma_\alpha - m^2\gamma_\nu \, . \end{aligned} \tag{B.3}$$

By direct computation of the above integral, one arrives exactly at the result found by the previous method, eq. (6.4). Indeed, the polarization operator calculated by these two methods turn out to be the same, eq. (6.5).

Now, we recalculate the diagram of the figure 2. Starting from (6.9), one has to perform the Feynman parametrization

$$\frac{1}{ab^2} = \int_0^1 \frac{2(1-x)dx}{(ax + (1-x)b)^3} \, , \tag{B.4}$$

and the usual variable shift $\bar{p} = p - xq$. After proper algebraic manipulations, one arrives at the expression:

$$I_{\nu\mu} = 2\int_0^1 dx(1-x) \int \frac{d^dp}{(2\pi)^d} \frac{(A_{\alpha\nu\beta\mu\rho} + A_{\alpha\nu\rho\mu\beta} + A_{\rho\nu\beta\mu\alpha})xq^\rho + B_{\alpha\nu\mu\beta}}{(p^2 - \Delta)^3} \, p^\alpha p^\beta \, . \tag{B.5}$$

Using known integrals in dimensional regularization, and performing the integration over $x$, we obtain

$$I_{\nu\mu} = \frac{i}{12\epsilon}(\gamma_\nu\gamma_\mu\gamma_\rho q^\rho + \gamma_\mu\gamma_\rho\gamma_\nu q^\rho + \gamma_\rho\gamma_\nu\gamma_\mu q^\rho) + \frac{1}{4\epsilon}(12m\eta_{\mu\nu} - 4m\gamma_\mu\gamma_\nu + 2q^\rho\gamma_\mu\gamma_\rho\gamma_\nu) + \cdots \tag{B.6}$$

and as we already have $J$, we arrive at a final result identical to (6.13).

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